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Dynamical P-parity Breaking in Effective Quark Model.

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Fermion models with selfinteraction including derivatives of fields are investigated in the strong coupling regime. The existence of three phases is established in the two channel model near tricritical point. The special phase of dynamical P-parity breaking is found.

1. Introduction.

Quark models with local four-fermion interaction are widely applied for the description of low-energy phenomena in the particle and nuclear physics [1]. In the extended version these models can be supplemented with quasilo-cal interaction, including derivatives of fermion fields [2]. For such models in the strong coupling regime the dynamical chiral symmetry breaking occurs and it results in the formation of dynamical quark mass as well as in the production of several meson states with the same quantum numbers [3] (so called "radial excitations" [4]).

It seemed to be the general feature for the quark models of Nambu-Jona-Lasinio type that, when being a symmetry of the quark lagrangian, the P-parity remains a good quantum number after the DCSB. However, it happens that for particular combination of four-fermion coupling constants in quasilocal vertices the P-parity can be broken dynamically together with the chiral symmetry.

In our talk we focus on composite boson states (mesons) created in scalar and pseudoscalar channels due to DCSB.

The two-channel model is examined in details near tricritical point and three major phases are described: a symmetrical one and two phases with

DCSB, different in correlation lengths in scalar channels. On a particular plane in the coupling constant space the complex solution for the dynamical mass function is found, which yields the (logarithmically suppressed) P-parity breaking in meson sector. It means that in such a phase there exist heavy scalar states which can decay into two or three pions.

2. Formulation of Quasilocal NJL-like Model.

We consider a class of effective fermion models with quasilocal interaction [3]. The lagrangian for a minimal model which allows the strong coupling regime in l -channels is following:

$$\mathcal{L}(x) = i\bar{\psi}\partial_\mu\gamma^\mu\psi + \frac{8\pi^2}{N_c\Lambda^2} \sum_{i,j=1}^l a_{ij}\bar{\psi}_L\varphi_i\left(-\frac{\partial^2}{\Lambda^2}\right)\psi_R\bar{\psi}_R\varphi_j\left(-\frac{\partial^2}{\Lambda^2}\right)\psi_L. \quad (1)$$

Here $\psi_{L(R)} \equiv 1/2(1 \pm \gamma_5)\psi$ are left and right spinors; a_{ij} is a non-degenerate real matrix of coupling constants. The singlet flavor fermionic fields in the model are transformed under $SU_c(N_c) \times U_F(1)$ group, where $N_c \gg 1$ is the number of colors, and $U_F(1)$ reflects the flavor symmetry. The sum over color indices is assumed. The formfactors $\varphi_i(\tau), (\tau \rightarrow -\partial^2/\Lambda^2)$ determine quasilocal interactions. We regularize the vertices by imposing the momentum cut-off $\Lambda : \bar{\psi}\psi \longrightarrow \bar{\psi}\theta(\Lambda^2 + \partial^2)\psi$, and choose the formfactors φ_i to implement

$$\int_0^1 d\tau \varphi_i(\tau)\varphi_j(\tau) = \delta_{ij}. \quad (2)$$

In order to develop the large- N_c approximation, it is convenient to express (1) in terms of auxiliary fields $\chi_i = \sigma_i + i\pi_i$:

$$\mathcal{L}_\chi(x) = i\sigma_j\bar{\psi}\varphi_j\psi - \pi_j\bar{\psi}\varphi_j\gamma_5\psi + \frac{N_c\Lambda^2}{8\pi^2}\chi_i^*a_{ij}^{-1}\chi_j. \quad (3)$$

Thereby, we come to a model with l scalar and pseudoscalar channels. When integrating out the fermionic fields, we obtain the effective action of χ, χ^* :

$$\exp(-S_{eff}(\chi)) = \langle \exp(-\int d^4x \mathcal{L}_\chi(x)) \rangle_{\psi, \bar{\psi}} \quad (4)$$

(in the Euclidian space-time).

The effective potential, $V_{eff} = S_{eff} \cdot (\text{vol})^{-1}$, proves to be a functional depending on the dynamical mass function $M(\tau) \equiv \chi_j \varphi_j(\tau)$ and proportional to N_c . It allows us to use the saddle point approximation for $N_c \gg 1$. The extrema of the effective potential can be found from mass-gap equation:

$$\frac{\delta V_{eff}}{\delta \chi_j^*} = 0. \quad (5)$$

The trivial solution $\chi_j = 0$ satisfies (5), and relates to the symmetrical phase. Non-trivial solutions bring a non-zero dynamical fermion mass $< M >_\chi \neq 0$. We study the model near the polycritical point, where $M \ll \Lambda$:

$$a_{ij} \sim \delta_{ij} + \frac{\Delta_{ij}}{\Lambda^2}, \quad |\Delta_{ij}| \ll \Lambda^2 \quad (6)$$

when the strong coupling regime occurs in each of the l -channels.

3. Two-Channel Model with DCSB.

A generalized NJL-like model with tricritical point is produced by two-channel interaction (1), when we set $i, j = 1, 2$ in (1)-(6). We retain only the lowest derivatives in the potential, with $\varphi_1 = 1$, $\varphi_2 = \sqrt{3}(1 - 2\tau)$. The dynamical mass function is thereby $M(\tau) = \bar{\chi}_1 + \bar{\chi}_2 \sqrt{3}(1 - 2\tau)$. As $\bar{\chi}_j$ are complex functions, $M(\tau)$ is complex too. However, with the global chiral rotation $M(\tau) \rightarrow M(\tau)e^{i\omega}$, $\omega = \text{const}$ it is always possible to implement $\text{Im} < M_0 >_\chi = 0$ and we can choose the following parametrization:

$$\bar{\chi}_1 = \chi_1 + i\rho, \quad \bar{\chi}_2 = \chi_2 - i\frac{\rho}{\sqrt{3}}, \quad \chi_i \equiv \text{Re} \bar{\chi}_i. \quad (7)$$

The equations (5) for the two-channel model read

$$\begin{aligned} \Delta_{11}\chi_1 + \Delta_{12}\chi_2 &= M_0^3 \ln \frac{\Lambda^2}{M_0^2} - 6\sqrt{3}\chi_1^2\chi_2 - 18\chi_1\chi_2^2 - 8\sqrt{3}\chi_2^3 \\ d_1\chi_1 - d_2\chi_2 &= 2\sqrt{3}\chi_1(\chi_1^2 + 3\chi_2^2) + 2\rho^2\left(\frac{4}{\sqrt{3}}\chi_1 - 2\chi_2\right) \\ \rho(\sqrt{3}\Delta_{11} - \Delta_{12}) &= 2\rho\sqrt{3}(\chi_1^2 + \chi_2^2 + \frac{4}{3}\rho^2), \end{aligned} \quad (8)$$

where

$$d_1 = \sqrt{3}\Delta_{11} - \Delta_{12}, \quad d_2 = -\sqrt{3}\Delta_{21} + \Delta_{22}. \quad (9)$$

We analyze the equations (8) near polycritical point, $|\Delta_{ij}| \sim \mu^2 \ll \Lambda^2$, in the large-log approximation ($\ln \frac{\Lambda^2}{\mu^2} \gg \ln \ln \frac{\Lambda^2}{\mu^2}$). It gives rise to a set of classes of solutions.

For $\rho = 0$ all the solutions are divided to the following classes:

a) Gross-Neveu-like solutions χ_j^{GN} :

$$\chi_1^2 = \frac{d_2^2 \det \Delta}{(\sqrt{3}d_1 + d_2)^3 \ln \frac{\Lambda^2}{\mu^2}} \left[1 + O \left(\frac{1}{\ln \frac{\Lambda^2}{\mu^2}} \right) \right], \quad \chi_2 \approx \frac{d_1}{d_2} \chi_1. \quad (10)$$

These solutions deliver minima to the potential when $\sqrt{3}d_1 + d_2 < 0$, with one eigenvalue of the matrix Δ being in the overcritical regime, the other in the undercritical.

b) Abnormal solutions are:

$$\chi_1^2 = \frac{\sqrt{3}d_1 + d_2}{12} \left[1 + O \left(\frac{1}{\ln^{1/3} \frac{\Lambda^2}{\mu^2}} \right) \right], \quad \chi_2 \approx -\frac{\chi_1}{\sqrt{3}}. \quad (11)$$

They give minima to the potential when $\sqrt{3}d_1 + d_2 > 0$, $\sqrt{3}d_1 - 2d_2 \neq 0$ (either both eigenvalues of Δ are positive, or one is positive and the other negative).

c) On the hyperplanes $\sqrt{3}d_1 + d_2 = 0$ and $\sqrt{3}d_1 - 2d_2 = 0$ there appear special solutions with a different asymptotics [3]. In general, in the models with more than one channel the complex solutions are allowed, and the imaginary parts of all the variables χ_j cannot be removed simultaneously by a global chiral rotation.

d) Complex solutions ($\rho \neq 0$) appear for those domains in vicinity of the hyperplane $\sqrt{3}d_1 - 2d_2 = 0$, their asymptotic expressions are:

$$\chi_1^2 = \frac{d_1 + 4\Delta_{12}}{16\sqrt{3}(\ln \frac{\Lambda^2}{\mu^2} - 3)} \quad \chi_2 \approx -\sqrt{3}\chi_1, \quad (12)$$

and the dynamical mass is $M_c^2 = 4\chi_1^2$. The axial part of the mass function looks as follows:

$$\rho^2 = \frac{d_1\sqrt{3}}{8} - \frac{3}{4}(\chi_1^2 + \chi_2^2) = \frac{d_1\sqrt{3}}{8} \left[1 + O \left(\frac{1}{\ln \frac{\Lambda^2}{\mu^2}} \right) \right]. \quad (13)$$

In each of the phase space's domains mentioned above one finds four common boson states — two scalar and two pseudoscalar — for real χ_j ,

and, in general, three states with mixed P-parity and one pseudoscalar with zero mass, the latest is in accordance to the Goldstone theorem. We discuss the spectrum of revealed states in the next section.

4. Second Variation and Mass Spectrum of Composite States.

The matrix of second variations of the effective potential determines the spectrum of bosonic states. We divide it on two parts: one independent on momentum, \hat{B} , and the kinetic part, which is proportional to momentum squared, $\hat{A}p^2$.

$$\delta^{(2)}S = (\delta\chi^*, (\hat{A}p^2 + \hat{B})\delta\chi) \quad (14)$$

$$\chi_j = \langle \chi_j \rangle + \delta\chi_j = \langle \chi_j \rangle + \sigma_j + i\pi_j. \quad (15)$$

The constant matrix \hat{B} has zero-mode $\chi_j^0 = \langle \pi_j \rangle - i \cdot \langle \sigma_j \rangle$, regarding to the existence of Goldstone bosons.

To find the spectrum of collective excitations one should solve the equation

$$\det(\hat{A}p^2 + \hat{B}) = 0. \quad (16)$$

Taking into account the conditions necessary for a minimum of the potential, we find the solutions at $-m^2 = p^2 \leq 0$, giving physical values of particle masses.

In the case of $\rho = 0$:

a) NJL-like mass spectrum:

$$\begin{aligned} m_\pi^2 = 0 \quad m_{\pi'}^2 &\approx m_{\sigma'}^2 \approx -\frac{\sqrt{3}d_1 + d_2}{3} \\ m_\sigma^2 &\approx 4M_0^2, \end{aligned} \quad (17)$$

In this domain the radial excitation states are heavier than the lightest scalar meson by a factor of logarithm.

b) For the abnormal solutions we have

$$\begin{aligned} m_\pi^2 = 0, \quad m_{\pi'}^2 &\approx \frac{1}{9} \left(\frac{4}{3}\right)^{1/3} \frac{(\sqrt{3}d_1 - 2d_2)^{4/3}}{(\sqrt{3}d_1 + d_2)^{1/3}} \frac{1}{\ln^{1/3} \frac{\Lambda^2}{\mu^2}}, \\ m_\sigma^2 &\approx 6M_{an}^2 \quad m_{\sigma'}^2 \approx \frac{2}{3}(\sqrt{3}d_1 + d_2). \end{aligned} \quad (18)$$

When comparing (17) and (18) we find the scalar channel correlation length to be different for each phase, that corresponds to the tricritical point conditions.

c) For the special real solutions the relations between scalar and pseudoscalar meson masses are different from (17),(18) (see [3]).

5. Mass Spectrum in the P-parity Breaking Phase.

One can see from (12),(13) that in the large-log approximation the axial dynamical mass (the imaginary part of $M(\tau)$) dominates. It leads to appearance of a massless boson in the scalar channel in accordance to the Goldstone theorem. Conventionally, the massless boson is interpreted as π -meson. For this purpose we make a global chiral rotation of fermionic fields $\psi \rightarrow \exp(i\gamma_5\pi/4)\psi$ accompanied by corresponding rotation of the bosonic variables $\bar{\chi}_j \rightarrow i\bar{\chi}_j$:

$$\bar{\chi}_1 = i\chi_1 - \rho, \quad \bar{\chi}_2 = i\chi_2 + \frac{\rho}{\sqrt{3}} \quad (19)$$

The classification of states given by P-parity quantum number is relevant only in the large-log approximation, when

$$\frac{B^{\pi\sigma}}{B^{\sigma\sigma}} \approx \frac{B^{\pi\sigma}}{B^{\pi\pi}} = O\left(\frac{1}{\ln \frac{\Lambda^2}{\mu^2}}\right) \quad (20)$$

next-to-leading logarithmic effects are of no importance and one can neglect mixing of the states with different P-parity. Then the spectrum of mesons is:

$$\begin{aligned} m_\pi^2 &= 0, & m_{\pi'}^2 &\approx \frac{d_1 + 4\Delta_{12}}{\sqrt{3} \ln \frac{\Lambda^2}{\mu^2}} \approx 16\chi_1^2 = 4M_c^2 \\ m_{\sigma'}^2 &\approx \sqrt{3}d_1, & m_\sigma^2 &\approx \frac{4(d_1 + \Delta_{12})}{9\sqrt{3} \ln \frac{\Lambda^2}{\mu^2}} \end{aligned} \quad (21)$$

The ratio of $m_{\pi'}$ and m_σ does not depend on logarithm, so both the masses are comparable. On the other hand, in the models with finite momentum cut-off, the effects of order of Λ^2 become sensible, the dynamical P-parity breaking is induced, since $B_{\pi\sigma} \neq 0$.

This phenomenon of dynamical P-parity breaking can be used in extensions of the Standard Model [5] where several Higgs bosons are composite ones. Thus we conclude that the models with polycritical points are drastically different from the local NJL models in the variety of the physical phenomena in the DCSB.

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